

MathCon

The Mathematics Firm

Bases Ortonormales

Ejercicios sobre bases Ortonormales

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Bases ortonormales en \mathbb{R}^2 .

Encontrar una base ortonormal a partir de la base dada.

1. $(3, 4), (-2, -2)$ Solución: $(\frac{3}{5}, \frac{4}{5}), (-\frac{4}{5}, \frac{3}{5})$
2. $(3, 4), (3, 0)$ Solución: $(\frac{3}{5}, \frac{4}{5}), (\frac{4}{5}, -\frac{3}{5})$
3. $(-6, -8), (-2, -10)$ Solución: $(-\frac{3}{5}, -\frac{4}{5}), (\frac{4}{5}, -\frac{3}{5})$
4. $(15, -8), (17, 13)$ Solución: $(\frac{15}{17}, -\frac{8}{17}), (\frac{8}{17}, \frac{15}{17})$
5. $(-15, -8), (-13, -6)$ Solución: $(-\frac{15}{17}, -\frac{8}{17}), (-\frac{8}{17}, \frac{15}{17})$
6. $(5, -12), (-3, -14)$ Solución: $(\frac{5}{13}, -\frac{12}{13}), (-\frac{12}{13}, -\frac{5}{13})$
7. $(3, -1), (3, 1)$ Solución: $(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}), (\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}})$
8. $(6, 3), (17, 5)$ Solución: $(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}), (\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}})$

9. $(17, 17), (9, -4)$

Solución: $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

10. $(-3, 2), (-2, -1)$

Solución: $(-\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}), (-\frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}})$

2

Bases ortonormales en \mathbb{R}^3 .

Encontrar una base ortonormal a partir de la base dada.

11. $(0, -1, -1), (-1, 1, 1)$ Solución: $(0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (-1, 0, 0)$
12. $(-2, -1, 2), (2, -3, 2)$ Solución: $(-\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}), (\frac{2}{3}, -\frac{2}{3}, \frac{1}{3})$
13. $(-2, -4, -4), (-2, 0, 1)$ Solución: $(-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}), (-\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}})$
14. $(-2, -1, -2), (0, 2, -1)$ Solución: $(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}), (0, \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}})$
15. $(1, 1, 0), (3, 1, -1)$ Solución: $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$
16. $(3, -1, -1), (3, 2, -1)$ Solución: $(\frac{3}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}), (\frac{3}{\sqrt{110}}, \sqrt{\frac{10}{11}}, -\frac{1}{\sqrt{110}})$
17. $(-5, -5, 5), (-3, 1, -2)$ Solución: $(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}), (-\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, -\sqrt{\frac{2}{7}})$

18. $(-4, 0, 4), (-2, 3, 0)$ Solución: $(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, -\frac{1}{\sqrt{11}})$
19. $(2, -5, -2), (4, 2, -1)$ Solución: $(\frac{2}{\sqrt{33}}, -\frac{5}{\sqrt{33}}, -\frac{2}{\sqrt{33}}), (\frac{4}{\sqrt{21}}, \frac{2}{\sqrt{21}}, -\frac{1}{\sqrt{21}})$
20. $(1, 3, 0), (3, -1, -3)$ Solución: $(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}, 0), (\frac{3}{\sqrt{19}}, -\frac{1}{\sqrt{19}}, -\frac{3}{\sqrt{19}})$
21. $(0, 0, 1), (0, -1, 1), (-1, 0, 1)$ Solución: $(0, 0, 1), (0, -1, 0), (-1, 0, 0)$
22. $(0, 1, 0), (0, -1, 1), (-2, 2, -1)$ Solución: $(0, 1, 0), (0, 0, 1), (-1, 0, 0)$
23. $(0, 0, 1), (-4, -3, 1), (4, 2, 2)$ Solución: $(0, 0, 1), (-\frac{4}{5}, -\frac{3}{5}, 0), (\frac{3}{5}, -\frac{4}{5}, 0)$
24. $(2, -1, -2), (-1, 1, 0), (2, -2, -2)$ Solución: $(\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}), (-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}), (-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3})$
25. $(0, -3, 3), (-2, 1, 2), (1, , -1, 2)$ Solución: $(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (-1, 0, 0), (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
26. $(0, -1, 1), (1, -1, 1), (1, 0, -1)$ Solución: $(0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (1, 0, 0), (0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$
27. $(-1, 0, -1), (-1, -1, -1), (-1, 0, 1)$ Solución: $(-\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}), (0, -1, 0), (-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$
28. $(-2, 0, 1), (2, 1, -1), (1, -2, 2)$ Solución: $(-\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}), (0, 1, 0), (\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}})$
29. $(-2, 0, 3), (-4, 0, -2), (-3, -2, -3)$ Solución: $(-\frac{2}{\sqrt{13}}, 0, \frac{3}{\sqrt{13}}), (-\frac{3}{\sqrt{13}}, 0, -\frac{2}{\sqrt{13}}), (0, -1, 0)$
30. $(2, -2, -3), (0, 4, -4), (4, -4, -4)$ Solución:
 $(\frac{2}{\sqrt{17}}, -\frac{2}{\sqrt{17}}, -\frac{3}{\sqrt{17}}), (-\frac{2}{\sqrt{561}}, \frac{19}{\sqrt{561}}, -\frac{14}{\sqrt{561}}), (-\frac{5}{\sqrt{33}}, \frac{2}{\sqrt{33}}, \frac{2}{\sqrt{33}})$