MathCon

The Mathematics Firm

Derivadas

Contenido

1 Derivadas

1.1. Derivadas directas

1. $f(x) = x^2 + x + 1$ R. $f'(x) = 2x + 1$.
2. $f(x) = x^a + x$ R. $f'(x) = ax^{a-1} + 1$.
3. $f(x) = (x^2 + x)(x)$ R. $f'(x) = 3x^2 + 2x$.
4. $f(x) = (x + 1)(x + 2)$ R. $f'(x) = 2x + 3$.
5. $f(x) = (x^2 - 1)(x^3 + 2)$ R. $f'(x) = 5x^4 - 3x^2 + 4x$.
6. $f(x) = (x^2 + x + 1)(x^3 - 2x + 3)$ R. $f'(x) = 5x^4 + 4x^3 - 3x^2 + 2x + 1$.
7. $f(x) = \sin^2(x)$ R. $f'(x) = \sin(2x)$.
8. $f(x) = x\cos(x)$ R. $f'(x) = \cos x - x\sin x$.
9. $f(x) = (\sqrt{x})(x^2 + 3)$ R. $f'(x) = \dfrac{3 + 5x^2}{2\sqrt{x}}$.
10. $f(x) = \sqrt[3]{x^2 + a}$ R. $f'(x) = \dfrac{2x}{3(a + x^2)^{(2/3)}}$.
11. $f(x) = a\sqrt{x + a}$ R. $f'(x) = \dfrac{a}{2\sqrt{a + x}}$.
12. $f(x) = (a^2 - x^2)\sqrt{x^2 + a^2}$ R. $f'(x) = -\dfrac{x(a^2 + 3x^2)}{\sqrt{a^2 + x^2}}$.
13. $f(x) = \sqrt{x^2 + b^2}\sqrt{x^2 + a^2}$ R. $f'(x) = \dfrac{x(a^2 + b^2 + 2x^2)}{\sqrt{a^2 + x^2}\sqrt{b^2 + x^2}}$.
14. $f(x) = (\sqrt{x} + 1)(\sqrt{x} - 2)$ R. $f'(x) = 1 - \dfrac{1}{2\sqrt{x}}$.
15. $f(x) = (\sqrt{x} + x)(\sqrt{x} - x)$ R. $f'(x) = 1 - 2x$.

16. $f(x) = (\sqrt{x} + x)/(\sqrt{x} - x)$ R. $f'(x) = \dfrac{1}{(\sqrt{x} - 1)^2\sqrt{x}}$.

17. $f(x) = 1/(\sqrt{x} + 1)$ R. $f'(x) = \dfrac{1}{2(\sqrt{x} + 1)^2\sqrt{x}}$.

18. $f(x) = (\sqrt{x} + 1)/(x^2 - x)$ R. $f'(x) = \dfrac{2 - 3\sqrt{x}}{2(\sqrt{x} - 1)^2x^2}$.

19. $f(x) = x/\sin x$ R. $f'(x) = (1 - x\cot x)(\csc(x))$.

20. $f(x) = \dfrac{\sin x}{1 - \cos x}$ R. $f'(x) = \dfrac{1}{\cos x - 1}$.

21. $f(x) = \sin x(\sin x + \cos x)$ R. $f'(x) = \cos 2x + \sin 2x$.

22. $f(x) = \dfrac{\sec x}{x}$ R. $f'(x) = \dfrac{\sec x(x\tan x - 1)}{x^2}$.

23. $f(x) = \dfrac{x}{1 - \sin x}$ R. $f'(x) = \dfrac{1 + x\cos x - \sin x}{(\sin x - 1)^2}$.

24. $f(x) = \dfrac{x}{a + bx^2}$ R. $f'(x) = \dfrac{a - bx^2}{(a + bx^2)^2}$.

25. $f(x) = 1/\sqrt{x}$ R. $f'(x) = -\dfrac{1}{2x\sqrt{x}}$.

26. $f(x) = x/\sqrt{x}$ R. $f'(x) = \dfrac{1}{2\sqrt{x}}$.

27. $f(x) = 1/\sqrt{x + 1}$ R. $f'(x) = -\dfrac{1}{2(1 + x)^{3/2}}$.

28. $f(x) = x/\sqrt{x^2 + 1}$ R. $f'(x) = \dfrac{1}{(1 + x^2)^{3/2}}$.

29. $f(x) = x^2/(x^3 + 1)$ R. $f'(x) = -\dfrac{x(x^3 - 1)}{(1 + x^3)^2}$.

30. $f(x) = (a - x)/(a + x)$ R. $f'(x) = -\dfrac{2a}{(a + x)^2}$.

31. $f(x) = \sin x\cos x$ R. $f'(x) = \cos(2x)$.

32. $f(x) = e^x(\sin x + \cos x)$ R. $f'(x) = 2e^x\cos x$.

33. $f(x) = \sin(\dfrac{1}{x})$ R. $f'(x) = -\dfrac{\cos(1/x)}{x^2}$.

34. $f(x) = 1/\sin x$ R. $f'(x) = -\cot x\csc x$.

35. $f(x) = 1/\sin(1/x)$ R. $f'(x) = \dfrac{\cot(1/x)\csc(1/x)}{x^2}$.

36. $f(x) = 1/\ln x$ R. $f'(x) = -\dfrac{1}{x\ln^2(x)}$.

37. $f(x) = x/\ln x$ R. $f'(x) = \dfrac{\ln x - 1}{\ln^2(x)}$.

38. $f(x) = \ln x/x$ R. $f'(x) = \dfrac{1-\ln x}{x^2}$.

39. $f(x) = x/(\sin^2(x)+x)$ R. $f'(x) = \dfrac{\sin(x)(\sin(x)-2x\cos(x))}{(\sin(x)^2+x)^2}$.

40. $f(x) = \tan x - x$ R. $f'(x) = \tan^2 x$.

41. $f(x) = (\sin(x)-\cos(x))/x$ R. $f'(x) = \dfrac{(x+1)\cos(x)+(x-1)\sin(x)}{x^2}$.

42. $f(x) = \dfrac{x^p}{x^m-a^m}$ R. $f'(x) = \dfrac{x^{p-1}(p-m)x^m - pa^m}{(x^m-a^m)^2}$.

43. $f(x) = \sqrt{x^2+a^2}$ R. $f'(x) = \dfrac{x}{x^2+a^2}$.

44. $f(x) = \sqrt{x+\sqrt{x}}$ R. $f'(x) = \dfrac{1+\dfrac{1}{2\sqrt{x}}}{2\sqrt{\sqrt{x}+x}}$.

45. $f(x) = \sqrt{\dfrac{x+1}{x}}$ R. $f'(x) = -\dfrac{1}{2\sqrt{1+1/x}x}$.

46. $f(x) = \sqrt{ax} + \dfrac{a}{\sqrt{ax}}$ R. $f'(x) = \dfrac{a}{2\sqrt{ax}} - \dfrac{a}{2x\sqrt{ax}}$.

47. $f(x) = x\ln x$ R. $f'(x) = \ln x + 1$.

48. $f(x) = \ln(\sqrt{x})$ R. $f'(x) = \dfrac{1}{2x}$.

49. $f(x) = \ln(1/\sqrt{x})$ R. $f'(x) = -\dfrac{1}{2x}$.

50. $f(x) = \ln(\ln x)$ R. $f'(x) = \dfrac{1}{x\ln x}$.

51. $f(x) = \sqrt{\dfrac{x+1}{x-1}}$ R. $f'(x) = -\dfrac{1}{(x-1)^2\sqrt{\dfrac{x+1}{x-1}}}$.

52. $f(x) = \sqrt{\dfrac{a-x}{a+x}}$ R. $f'(x) = -\dfrac{a}{(a+x)^2\sqrt{\dfrac{a-x}{a+x}}}$.

53. $f(x) = \sqrt{\dfrac{a^2+x^2}{a^2-x^2}}$ R. $f'(x) = \dfrac{2a^2x}{(a^2-x^2)\sqrt{a^4-x^4}}$.

54. $f(x) = \sqrt{\dfrac{\sin(x)}{\cos(x)}}$ R. $f'(x) = \dfrac{\sec^2(x)}{2\sqrt{\tan(x)}}$.

55. $f(x) = \sin(x^2)\cos^2(x)$ R. $f'(x) = 2\cos^{2x}(x)\cos(x^2)$.

56. $f(x) = \sin(\sqrt{x})/x$ R. $f'(x) = \dfrac{\sqrt{x}\cos(\sqrt{x}) - 2\sin(\sqrt{x})}{2x^2}$.

57. $f(x) = e^{\sqrt{x}}$ R. $f'(x) = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$.

58. $f(x) = \dfrac{e^x - e^{-x}}{2}$ R. $f'(x) = \dfrac{e^x + e^{-x}}{2}$.

59. $f(x) = \ln(1/x)$ R. $f'(x) = -1/x$.

60. $f(x) = \ln x \ln x$ R. $f'(x) = \dfrac{2\ln x}{x}$.

61. $f(x) = \ln(\ln(\ln(1/x)))$ R. $f'(x) = -\dfrac{1}{x\ln(1/x)\ln(\ln(1/x))}$.

62. $f(x) = \arcsin(x/a)$ R. $f'(x) = \dfrac{1}{\sqrt{a^2 - x^2}}$.

63. $f(x) = e^{e^x}$ R. $f'(x) = e^{e^x + x}$.

64. $f(x) = x^x$ R. $f'(x) = x^x(1 + \ln x)$.

65. $f(x) = x^{x^x}$ R. $f'(x) = x^{x^x + x - 1}(1 + x\ln x + x\ln^2(x))$.

66. $f(x) = 2^{3^x}$ R. $f'(x) = 2^{3^x} 3^x \ln 2 \ln 3$.

67. $f(x) = 2^{3^{5^x}}$ R. $f'(x) = 2^{3^{5^x}} 3^{5^x} 5^x \ln 2 \ln 3 \ln 5$.

68. $f(x) = e^x \ln x$ R. $f'(x) = \dfrac{e^x(1 + x\ln x)}{x}$.

69. $f(x) = e^{1/x}$ R. $f'(x) = -\dfrac{e^{1/x}}{x^2}$.

70. $f(x) = e^{1/x} e^x$ R. $f'(x) = e^{1/x + x}(1 - \dfrac{1}{x^2})$.

71. $f(x) = e^{\dfrac{a+bx}{a-bx}}$ R. $f'(x) = \dfrac{2abe^{\dfrac{a+bx}{a-bx}}}{(a-bx)^2}$.

72. $f(x) = e^{\sin x}$ R. $f'(x) = e^{\sin x}\cos x$.

73. $f(x) = e^x \sin x$ R. $f'(x) = e^x(\cos x + \sin x)$.

74. $f(x) = a^{\tan nx}$ R. $f'(x) = na^{\tan nx}\ln(a)\sec^2(nx)$.

75. $f(x) = x^{1/x}$ R. $f'(x) = x^{1/x}(1 - \ln x)/x^2$.

76. $f(x) = x^{\sqrt{x}}$ R. $f'(x) = \dfrac{x^{\sqrt{x}}(2 + \ln x)}{2\sqrt{x}}$.

77. $f(x) = \sin^x x$ R. $f'(x) = \sin^x x(x\cot x + \ln(\sin x))$.

78. $f(x) = \sin^{\tan x} x$ R. $f'(x) = \sin^{\tan x} x(1 + x\sec^2 x\ln(\sin x))$.

79. $f(x) = \sin^{\tan x} x$ R. $f'(x) = \sin^{\tan x} x(1 + x\sec^2 x\ln(\sin x))$.

80. $f(x) = \sqrt{\frac{\sin x + 1}{\sin x - 1}}$ R. $f'(x) = -\dfrac{\cos x}{(\sin x - 1)^2\sqrt{\frac{\sin x + 1}{\sin x - 1}}}$.

81. $f(x) = \arcsin\sqrt{\sin x}$ R. $f'(x) = \dfrac{\cos x}{2\sqrt{\sin x - \sin^2 x}}$.

82. $f(x) = e^{\arctan x}$ R. $f'(x) = \dfrac{e^{\arctan x}}{1+x^2}$.

83. $f(x) = \arctan\left((e^x - e^{-x})/2\right)$ R. $f'(x) = \dfrac{2}{e^x + e^{-x}}$.

84. $f(x) = \sqrt{x + \sqrt{1+x^2}}$ R. $f'(x) = \dfrac{\sqrt{x+\sqrt{1+x^2}}}{2\sqrt{1+x^2}}$.

85. $f(x) = \sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}$ R. $f'(x) = -\dfrac{1}{2(1+\sqrt{x})(\sqrt{x(1-x)})}$.

86. $f(x) = \sin^m x \cos^n x$ R. $f'(x) = \sin^{m-1} x \cos^{n-1} x(m\cos^2 x - n\sin^2 x)$.

87. $f(x) = \cot^2(\sin x)$ R. $f'(x) = -\cos x \cot(\sin x)\csc^2(\sin x)$.

88. $f(x) = \sqrt{x\sqrt{x}}$ R. $f'(x) = \dfrac{3\sqrt{x^{3/2}}}{4x}$.

89. $f(x) = \sqrt{1/x + \sqrt{1/x}}$ R. $f'(x) = \dfrac{-2 - \dfrac{1}{\sqrt{1/x}}}{4\sqrt{\sqrt{1/x} + 1/x}x^2}$.

90. $f(x) = \sqrt{x + \sqrt{x+\sqrt{x}}}$ R. $f'(x) = \dfrac{1 + \dfrac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}$.

91. $f(x) = \sqrt{2+\sqrt{2+\sqrt{x}}}$ R. $f'(x) = \dfrac{1}{8\sqrt{2+\sqrt{2+\sqrt{x}}}\sqrt{2+\sqrt{x}}\sqrt{x}}$.

92. $f(x) = \ln\left(\dfrac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+x}\right)$ R. $f'(x) = \dfrac{-2}{\sqrt{x^2+1}}$.

93. $f(x) = x^n(a+bx)^m$ R. $f'(x) = (x^n(a+bx)^m)\left(\dfrac{n}{x} + \dfrac{mb}{a+bx}\right)$.

94. $f(x) = \sin(\sqrt{x}) + \sqrt{\sin(x)}$ R. $f'(x) = 1/2\left(\dfrac{\cos(\sqrt{x})}{\sqrt{x}} + \dfrac{\cos(x)}{\sqrt{\sin(x)}}\right)$.

95. $f(x) = \sin(nx)\cos(mx)$ R. $f'(x) = n\cos(mx)\cos(nx) - m\sin(mx)\sin(nx)$.

96. $f(x) = \tan(nx)\cot(mx)$ R. $f'(x) = n\cot(mx)\sec^2(nx) - m\csc^2(mx)\tan(nx)$.

97. $f(x) = e^{nx}\ln(mx)$ R. $f'(x) = \dfrac{e^{nx}(1+nx\ln(mx))}{x}$.

98. $f(x) = 2^x 3^x 5^x$ R. $f'(x) = 30^x \ln(30)$.

99. $f(x) = 3^{\sqrt{x}}5^{\sqrt{x}}$ R. $f'(x) = -\dfrac{(3/5)^{\sqrt{x}}\ln(5/3)}{2\sqrt{x}}$.

100. $f(x) = \sqrt{\dfrac{\sin(e^{2x})}{\ln(\sqrt{x})}}$

R. $f'(x) = \dfrac{2e^{2x}\cos(e^{2x})\ln(x) - \sin(e^{2x})}{\sqrt{2}x\ln^2 x\sqrt{\dfrac{\sin(e^{2x})}{\ln x}}}$.

1.2. Derivadas implícitas